Understanding Robustness of Mobile Networks through Temporal Network Measures

Salvatore Scellato, Ilias Leontiadis, Cecilia Mascolo
University of Cambridge
Email: name.surname@cl.cam.ac.uk

Prithwish Basu
BBN Technologies
Email: pbasu@bbn.com

Murtaza Zafer
IBM T. J. Watson Research
Email: mzafer@us.ibm.com

***Abstract*—The application of complex network theory to communication systems has led to several important results. Nonetheless, previous research has often neglected to take into account their temporal properties, which in many real scenarios play a pivotal role. Mainly because of mobility, transmission delays or protocol design, a communication network should not be considered only as a static entity. At the same time, network robustness has come extensively under scrutiny. Understanding whether networked systems can undergo structural damage and yet perform efficiently is crucial to both their protection against failures and to the design of new applications. In spite of this, it is still unclear what type of resilience we may expect in a network that continuously changes over time.**

In this work we present the first attempt to define the concept of temporal network robustness: we describe a measure of network robustness for time-varying networks and we show how it performs on different classes of random models by means of analytical and numerical evaluation. Particularly, we show how static approximation can wrongly indicate high robustness of fragile networks when adopted in mobile time-varying networks, while a temporal approach captures more accurately the system performance.

I. Introduction

The study of real-world communication systems by means of complex network models has provided insightful results and has vastly expanded our knowledge on how single entities create connections and how these connections are used for communication or, more generally, interaction [1]. In particular, technological networks such as the Internet and the World Wide Web have been under scrutiny in terms of structure and dynamic behavior [2]. More recently, with the widespread adoption of mobile and opportunistic networks, it has become important to develop new analytical tools to keep into account network dynamics over time [3], [4], [5] and how this affects phenomena such as information propagation [6], [7]. Results have shown that time correlations and relative temporal ordering of connection events among nodes cannot be neglected, otherwise the performance of a given system can be greatly overestimated [3], [4].

At the same time, the problem of understanding whether real systems can sustain substantial damage and still maintain acceptable performance has been extensively addressed [8]. Various measures of network robustness have been defined and investigated for several classes of networks, evaluating how different system can be more or less resilient against random errors or targeted attacks thanks to their underlying structural properties [9], [10].

Nonetheless, it is still unclear how to approach the study of robustness of networks by taking into account their time-varying nature: by adopting a static representation of a temporal network, important features that impact the actual performance might be missed. Thus, it becomes important to develop a robustness metric that takes into account the temporal dimension and gives insights on how a mobile network is affected by damage or change. Particularly, the fact that links are not always active means that information spreading can be delayed or even stopped and that *relative ordering in time of connection events may affect the creation of temporal paths in mobile networks*.

Our main goal is to design a novel framework for the analysis of robustness in mobile time-varying networks. We adopt *temporal network metrics* [3] to quantify network performance and define a measure of robustness against generic network damages. We study its performance on random network models to understand its properties, describing how temporal robustness gives a more accurate evaluation of system resilience than static approaches.

Our contributions can be summarized as follows:

- We describe the use of temporal network metrics such as temporal distance to estimate the current network connectivity taking temporal variability into account. We define *temporal network robustness* (Section II), a novel measure that quantifies how the communication of a given time-varying network is affected by damage.
- We evaluate temporal robustness through numerical simulation both on a temporal version of the Erdős Rényi (ER) random graph model, on a Markov-based link connectivity model, investigating the effect of time-correlations, and on two random mobility models, which instead introduce space-correlations (Section III).
- We show how *temporal networks do not exhibit sharp breakdowns but instead fail gracefully when they are subjected to failures*. The temporal dimension is able to capture the evolution dynamics, exposing the fact that time allows to create temporal paths across otherwise disconnected portions of the network.

We discuss some implications of our findings for the design of new systems and applications (Section IV). Finally, we review related results on this topic (Section V) and we conclude the paper (Section VI).

II. Temporal Robustness

In this section, we will review some basic metrics for temporal networks and describe how these measures can be adopted to quantitatively define temporal network robustness.

A. Network Robustness

The study of robustness of complex networks has mainly focused on describing how a given performance metric of the network is affected when nodes are removed according to a certain rule. The underlying assumption is that the absence or malfunctioning of some nodes will cause the removal of their edges and, thus, some paths will become longer, increasing the distances between the remaining nodes, or completely disappear, resulting in the loss of connectivity in the whole system. The performance measures previously adopted include the network diameter [8], the size of the giant component [8], [9] and the average inverse geodesic length [9], [10]. Moreover, the strategies used to choose which nodes are to be removed can be divided in two broad categories: random failures, where every node has the same probability of being removed, and targeted attacks, where nodes are ranked according to a performance metric and then accordingly removed [9].

In this work, we will study the problem of defining and analyzing robustness in evolving networks: as a consequence, *we need to use a performance metric that includes the temporal dimension in its definition*. At the same time, we focus on the first strategy of node removal: we consider random and independent failures for every node and we evaluate how the system tolerates increasing level of malfunctioning nodes.

B. Temporal Network Metrics

In networks where nodes can connect with a large number of other nodes over time (e.g., mobile and peer to peer networks) time ordering of events is important. Traditional approaches that aggregate links over time always overestimate network connectivity, which means that some paths that seem to be present are in reality not present due to the actual ordering of the links over time. Hence, in this section we describe the notion of *temporal graph* we use to model mobile networks [3], [4].

Temporal graph: A temporal graph is a graph $G(t) = (V(t), E(t))$ where $V(t)$ is the set of nodes at time t and $E(t)$ is the set of edges at time t. We assume that $|V(t)| = N$, thus nodes cannot be added or removed from the graph. Furthermore, we treat time as a discrete entity and we create a sequence of graphs $G(t_1), \dots G(t_2)$ by adopting an appropriate resolution time.

Temporal distance: Then, given two nodes i and j we can define a temporal path $p_{ij}(t_1, t_2)$ between them in the time window $[t_1, t_2]$. The length of a temporal path is defined as the amount of time steps it takes to spread information from node i to node j on that particular path. This value is always a positive integer. As a consequence, we can define the *shortest temporal distance* $d_{ij}(t_1, t_2)$ as the smallest length among all the temporal paths between nodes i and j in time window $[t_1, t_2]$. For example, if a message sent by node A is received by node B at time t_d, then $d_{AB} = t_d - t_1$. If there is no temporal path between i and j in $[t_1, t_2]$, their distance can be considered infinite, i.e. $d_{ij}(t_1, t_2) = \infty$. The *average temporal distance* $L(t_1, t_2)$ of a given temporal graph G during a time window $[t_1, t_2]$ is defined as:

$$L(t_1, t_2) = \frac{1}{N(N-1)} \sum_{i,j} d_{ij}(t_1, t_2) \tag{1}$$

This quantity is not well defined when some pairs of nodes are not temporally connected. As a consequence, another metric has been introduced.

Temporal efficiency: We can define the *temporal global efficiency* $E(t_1, t_2)$ of a given temporal graph G

$$E(t_1, t_2) = \frac{1}{N(N-1)} \sum_{i,j} \frac{1}{d_{ij}(t_1, t_2)} \tag{2}$$

This value is not affected by disconnected pairs of nodes, because their contribution to the efficiency is computed as zero. Network efficiency is normalized between 0 and 1 and it does not depend on the size of network, hence, it can be adopted to compare graphs with different sizes.

Since a temporal graph is continuously evolving, we can evaluate how temporal efficiency changes over time by considering a value τ and evaluating $E_G(t)$ as the relative temporal efficiency of the temporal graph in the time window $[t - \tau, t]$. Similarly, $L_G(t)$ is computed as the average temporal distance in $[t - \tau, t]$. If the properties of the temporal graph are stationary, we expect these values to reach a steady value as time progresses.

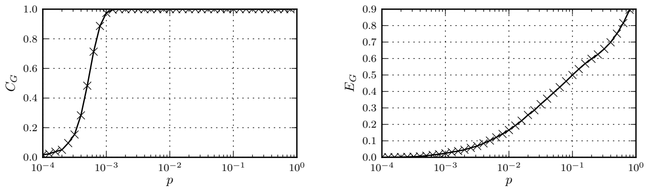
C. Temporal Robustness Metric

Given a temporal graph G, we define a damage D as any structural modification on it and we define G_D as the graph resulting by the effect of the damage D on G. A damage may be the deactivation of some nodes or the removal of some edges at a particular time t_D. Because of damage D, some temporal shortest paths will be longer or will not exist any more, thus, we expect that the temporal efficiency will eventually reach a new steady value $E_{G_D} \leq E_G$. It is important to evaluate the new value of the temporal efficiency on a new temporal graph that still contains the deactivated nodes, in order to obtain a decrease in efficiency. Otherwise, we might obtain a smaller temporal graph that is more efficient than the original graph, although it has lost much of its structure.

We define the loss in efficiency $\Delta E(G, D)$ caused by the damage D on the temporal graph G as $\Delta E(G, D) = E_G - E_{G_D}$. Finally, we define the **temporal robustness** $R_G(D)$ of the temporal graph against the damage D as

$$R_G(D) = 1 - \frac{\Delta E(G, D)}{E_G} = \frac{E_{G_D}}{E_G} \tag{3}$$

This value is normalized between 0 and 1 and it measures the relative loss of efficiency caused by the damage: if the damage does not effect the efficiency of the graph ($E_{G_D} = E_G$) then its robustness is 1, while if the damage destroys the efficiency of the graph ($E_{G_D} = 0$) the robustness drops to 0.



(a) Temporal connected couples (b) Temporal efficiency

Fig. 1. Fraction of connected couples of nodes C_G (a) and temporal efficiency E_G (b) as a function of link probability p for a temporal ER graph with $N = 100$ nodes and with $T = 100$. Simulation results are averaged over 100 runs.

III. Numerical Evaluation

In this section, we present a numerical analysis of temporal metrics and robustness for different classes of random temporal networks.

A. Temporal Network Models

Here we describe the different temporal network models we investigate in this work.

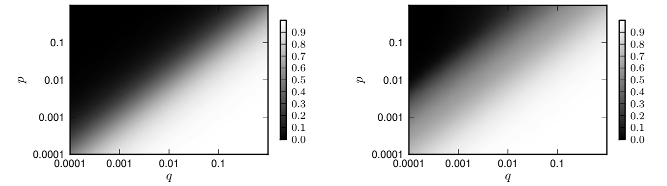
1) Erdős-Rényi Temporal Network Model: An Erdős-Rényi (ER) random graph with N nodes and parameter p is created by independently including each possible edge in the graph with probability p and it is denoted as $G(N, p)$ [11]. We generalize this model to the temporal case by creating a sequence of T ER random graphs $G(N, p)$ and we denote the resulting temporal graph as $G(N, p, T)$.

2) Markov-based Temporal Network Model: The temporal ER network model does not provide temporal correlations between consecutive graphs in the sequence. We now consider a model where link evolution is described by a Markov process, thus enabling memory effects in network dynamics.

We consider a complete graph G with N nodes. At every discrete timestep t each link may or may not be present: a temporal graph is created where the existence of each link evolves according to a 2-state discrete Markov process. We denote with p the probability that a link present at time t will be removed at time $t+1$ and with q the probability that a link which is not present at time t will be added at time $t + 1$. The steady probability of link presence then is $P_{ON} = \frac{q}{p+q}$: as a consequence, each observation of the temporal graph appears as an ER random graph with each edge present with probability P_{ON}.

3) Mobility-based Temporal Network Models: We can create a random model of a temporal network by using mobility models. In this case we are introducing topological constraints: a key difference with the previous temporal models is that each node is not equally likely to connect with all the other nodes, due to the effect of spatial distance.

We consider $N = 100$ nodes moving in a square area $1000\text{x}1000$ meters and we define a communication range r: at every time step, we create a graph where nodes are connected if their Euclidean distance is shorter than r. Thus, we change the probability of link presence P_{ON} by varying the communication range. Then, a temporal graph can be defined as the sequence of graphs extracted at each time step while the nodes move. We investigate two different mobility models that are implemented using the Universal Mobility Model Framework [12]: Random Waypoint Model (RWP) and Random Waypoint Group Model (RWPG).



(a) P_{ON} (b) E_G

Fig. 2. Probability of link presence P_{ON} (a) and temporal efficiency E_G (b) as a function of parameters p and q in the Markov-based temporal network model. The two quantities exhibit similar trends in the parameter space.

In RWP each node selects uniformly at random a location towards which it moves with speed uniformly distributed in a fixed range $[5, 40]$ mph. As the node reaches its destination, it waits for a randomly distributed time in $[0, 120]$ seconds and repeats the above steps until the end of the simulation.

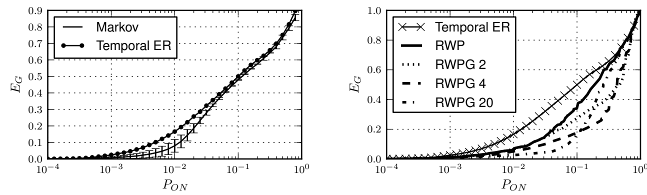
In RWPG nodes are divided into two classes: there are M group leaders and $N - M$ group followers. Every group followers has its own leader so that the N nodes are divided into equally-sized groups. Each group leader selects a random target and moves towards it, similarly to the RWP mobility model. Group members do not select any target; instead, they follow their group leader according to the *pursuit force* [12] which is set to give a group span of 200 meters.

B. Simulation Strategy

We numerically evaluate temporal efficiency $E_G(t)$ over time, adopting a time window of $\tau = 100$, for a graph with $N = 100$ nodes: after an initial phase, the random temporal graph reaches an equilibrium state and we compute the steady value of temporal efficiency. We run each simulation for 2τ steps and we compute the average value of temporal efficiency over the last τ steps. All results have been averaged over 100 different runs. We evaluated numerically temporal robustness by deactivating each node independently with incresing probability P_{ERR}. We measure temporal efficiency before and after the failure, when the network reaches a new equilibrium state.

C. Temporal Metrics

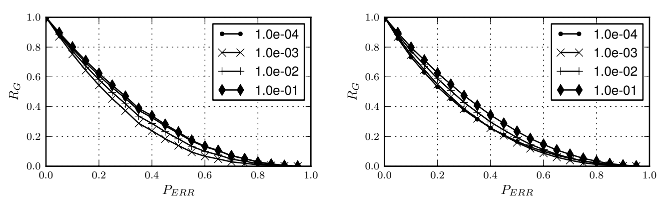
1) ER-based model: As we see in Figure 1, as the probability p increases both temporal efficiency and the number of connected couples increases. Furthermore, *we note that there is no evidence of sharp transition from a disconnected to a connected temporal graph*, since as we increase p the fraction of connected couples C_G smoothly increases from 0 to 1. This effect is due to the temporal dimension: in this model, no matter how small p is, the temporal graph will be eventually connected as $T \to \infty$ as long as there is a non-zero probability that each link is present. On the other hand, in the static case a ER random graph will experience a sharp transition and will be connected, on average, only when p approaches $\frac{\ln N}{N}$ [11].



(a) (b)

Fig. 3. Left: Temporal efficiency E_G as a function of probability of link presence P_{ON} in the Markov model and compared to the temporal ER model. The Markov model has error bars which show standard deviation of E_G for different parameter combinations which hold approximately the same P_{ON} (logarithmic binning has been adopted). Right: Temporal efficiency E_G as a function of probability of link presence P_{ON} for different mobility-based network models: Random Waypoint Model (RWP) and Random Waypoint Group Model (RWPG) with different number of groups. Temporal ER model is shown for comparison.

2) Markov-based model: Figure 2(a) reports the probability of link presence P_{ON} as a function of the two parameters of the Markov process p and q: the parameter space appears divided in two regions according to which parameter is larger than the other. As shown in Figure 2(b), temporal efficiency shows a similar behavior as P_{ON} in the parameter space. This is an indication that in this model the most important parameters is the probability of link presence.

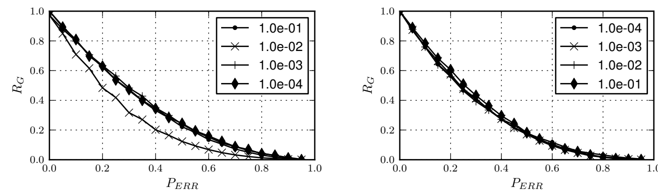


(a) $N = 100$ (b) $N = 1000$

Fig. 4. Temporal robustness R_G as a function of probability of error P_{ERR} in the ER random model for different link probability p. The size of the system has no impact on temporal robustness: furthermore, the system fails smoothly as the probability of error increases.

This intuition is only partially confirmed in Figure 3(a), where temporal efficiency is a function of the probability of link presence both for the temporal ER model and for the Markov model. Similar values of P_{ON} results in similar values of efficiency, regardless the actual values of p and q. Yet, the same value of P_{ON} results, on average, in higher efficiency in the temporal ER case, since avoiding time-correlations allows the creation of new edges at higher rate: thus, given an equal time interval, single nodes have more opportunities to communicate directly with new nodes in the uncorrelated case. Instead, for higher values of P_{ON} the two models behave in a similar way as they reach almost complete connectivity: this is because with high P_{ON} at each time step every node is connected by a direct link to a large fraction of the other nodes and so efficiency is already close to the maximum value.

3) Mobility-based models: Figure 3(b) depicts temporal efficiency E_G as a function of probability of link presence in the RWP case (estimated P_{ON}). There is a trend similar to the previous models: however, for the same value of P_{ON},



(a) Markov (b) RWP

Fig. 5. Temporal robustness R_G as a function of probability of error P_{ERR} and for different values of P_{ON} for the Markov-based and for the RWP random model (RWPG does not deviate from RWP)

the resulting efficiency is always smaller in the RWP case than in the temporal ER case because nodes move in a geographically restricted manner and, thus, they do not exhibit the same probability to connect with any other node. This is an important observation: in more realistic mobile scenarios, efficiency might be affected by spatial correlations.

We also investigated RWPG with various group sizes and we present here three extreme situations: i) 20 groups of 5 nodes (RWPG 20), ii) 4 groups of 25 nodes (RWPG 4) and iii) 2 groups of 50 nodes (RWPG 2). Figure 3(b) presents the temporal efficiency E_G obtained for the RWPG case: in this model, the spatial distribution of the groups appears to have a major impact on the efficiency. Group mobility is less efficient than RWP: group members have high efficiency between them but much smaller efficiency with nodes that belong to other groups. Moreover, every RWPG scenario undergoes a transition in the trend of E_G: as P_{ON} increases, there is a particular value when the efficiency starts increasing more quickly, as different groups finally become in direct temporal connection with each other, rather than connected through longer temporal paths. Before this transition, communication mainly occurs within single groups, so larger groups result in higher temporal efficiency. After this transition, scenarios with smaller groups become more efficient, since they enjoy very fast communication both within and among groups.

D. Temporal Robustness

1) ER-based model: As reported in Figure 4, the temporal ER model fails smoothly as we increase the fraction of removed nodes, without any sudden disruption for any value of P_{ERR}. This is a main difference with respect to what happens in the static case: for a static ER random graph be a critical value of P_{ERR} which causes a breakdown of the network in several disconnected components may exist [8]. This is not true for temporal robustness, as new paths can still appear after the damage as the network rearranges its connections. Time provides more redundancy and, hence, more resilience. Moreover, we also note that temporal robustness does not depend on system size: since it is normalized with respect to the value of temporal efficiency before the damage, it depends only on the relative drop in efficiency, not on absolute values.

2) Markov-based Model: As shown in Figure 5(a), temporal robustness is affected by probability of error P_{ERR} in the same way as in the temporal ER model: the system fails gradually as more nodes are removed. However, for intermediate values of P_{ON} robustness has lower values. Figure 3(a) depicts how exactly in the same range of P_{ON} the Markov-model deviates from the temporal ER: this indicates how in that range of values memory effects result in a network which is not well connected nor highly dynamic, with consequently lower values of temporal efficiency. At the same time, high and low values of P_{ON} provide the same robustness, even if the absolute value of temporal efficiency can be very different, thanks to the normalization of temporal robustness.

3) Mobility-based Models: In the case of mobility-based temporal networks, reported in Figure 5(b), both RWP and RWPG exhibit a similar behavior: again, the network loses efficiency in a smooth way and temporal robustness is not affected by P_{ON} in this case as the spatial characteristics of the network are mainly affecting the resulting robustness.

IV. IMPLICATIONS

In the previous sections we have seen how static robustness is not adequate to capture all aspects of mobile networks: instead, a temporal approach allows for a better understanding of the robustness, since it takes into account time-dependant connections. Our work presents many implications for the study of mobile networks and for the design of systems and applications in this domain.

First of all, a key advantage of our approach is that temporal robustness accurately models connectivity disruption in mobile networks: random models fail in a controlled way as we increase the fraction of removed nodes, without any sudden network disconnection. Another important property of the approach is that it does not overestimate connectivity. Time ordering and the temporal connectivity threshold τ exclude a number of connection paths that the static analysis would include. Therefore, the temporal model is able to correctly identify network connectivity disruptions, especially in real networks, where time ordering is important.

Finally, this approach can be implemented in a real mobile network: each node i may maintain a table of *Lamport clocks* [13] that contains the shortest temporal distances to all known nodes that can contact i within τ time steps. Clocks are then updated whenever two nodes get in contact.

V. RELATED WORK

One of the first attempts to generalize static network models to handle temporal information was to adopt time labels on edges to express temporal constraints on their presence [5]: this mainly algorithmic approach does not handle temporally disconnected nodes and, thus, is less suitable to investigate temporal networks arising from communication systems. Instead, some first attempts to investigate the properties of human contact networks reported on the temporal correlations and periodicities in these systems that arise at peculiar time scales [6]. More recently, the concept of network distance for temporal graphs has been formalized and explored [3], [4]. We build on these results, by adopting these temporal measures in the definition of temporal network robustness.

Nonetheless, there have been no attempts to address the problem of network robustness in time-varying graphs. To our knowledge, our work presents the first method to quantitatively evaluate network robustness by taking into account temporal properties.

VI. CONCLUSION

This paper has presented a study of temporal robustness in time-varying network: we adopt temporal network metrics to assess network performance in presence of increasingly larger random failures. We have investigated the performance of our method via simulations in different random models, exploring how the temporal dimension provides more redundancy to communication systems compared to static evaluation. We plan to extend our work by studying temporal robustness against attacks on random models and real networks.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Army Research Laboratory and the U.K. MOD under Agreement Number W911NF-06-3-0001. The views and conclusions contained in this document are those of the author(s) and should not be interpreted as representing the official policies, either expressed or implied, of the U.S. Army Research Laboratory, the U.S. Government, the U.K. MOD or the U.K. Government. The U.S. and U.K. Governments are authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation hereon.

REFERENCES

[1] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D. Hwang, "Complex networks: Structure and dynamics," *Physics Reports*, vol. 424, no. 4-5, pp. 175–308, Feb. 2006.

[2] M. Faloutsos, P. Faloutsos, and C. Faloutsos, "On power-law relationships of the Internet topology," in *Proceedings of SIGCOMM '99*. New York, NY, USA: ACM, 1999, pp. 251–262.

[3] J. Tang, M. Musolesi, C. Mascolo, and V. Latora, "Temporal Distance Metrics for Social Network Analysis," in *Proceedings of WOSN '09*, Barcelona, Spain, August 2009.

[4] J. Tang, S. Scellato, M. Musolesi, C. Mascolo, and V. Latora, "Small-world behavior in time-varying graphs," *Phys. Rev. E*, vol. 81, no. 5, p. 055101, May 2010.

[5] D. Kempe, J. Kleinberg, and A. Kumar, "Connectivity and inference problems for temporal networks," in *J. Comput. Syst. Sci*, 2000, p. 2002.

[6] A. Clauset and N. Eagle, "Persistence and Periodicity in a Dynamic Proximity Network," in *Proceedings of DIMACS Workshop on Computational Methods for Dynamic Interaction Networks*, September 2007.

[7] A. Chaintreau, P. Hui, J. Crowcroft, C. Diot, R. Gass, and J. Scott, "Impact of Human Mobility on Opportunistic Forwarding Algorithms," *IEEE Trans. on Mobile Computing*, vol. 6, no. 6, pp. 606–620, 2007.

[8] R. Albert, H. Jeong, and A.-L. Barabási, "Error and attack tolerance of complex networks," *Nature*, vol. 406, no. 6794, pp. 378–382, July 2000.

[9] P. Holme, B. J. Kim, C. N. Yoon, and S. K. Han, "Attack vulnerability of complex networks," *Phys. Rev. E*, vol. 65, no. 5, May 2002.

[10] P. Crucitti, V. Latora, M. Marchiori, and A. Rapisarda, "Error and attack tolerance of complex networks," *Physica A*, vol. 340, pp. 388–394, 2004.

[11] P. Erdős and A. Rényi, "On the evolution of random graphs," *Publications of the Mathematical Institute of the Hungarian Academy of Sciences*, vol. 5, pp. 17–61, 1960.

[12] A. Medina, G. Gursun, P. Basu, and I. Matta, "On the Universal Generation of Mobility Models," in *Proceedings of IEEE/ACM MASCOTS '10*, Miami Beach, FL, US, August 2010.

[13] L. Lamport, "Time, Clocks, and the Ordering of Events in a Distributed System," *Commun. ACM*, vol. 21, no. 7, pp. 558–565, 1978.